Orientation interpolation and applications

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ABSTRACT

Psychovision have shown that many grouping laws come into play to structure human vision. They use informations of different kinds, not only gray (or color)-level values. Here we will show how an orientation interpolation operator working in S^1 (angle in $[0, 2\pi[$) can be used to recover geometrical information in images. The operator is presented and is used to produce fields that drive a Fast Marching contour extraction algorithm and a LIC-based smoothing method. Experiment on real images are reported to validate the proposed approach.

1. INTRODUCTION

The human vision is known for its ability to use global and heterogeneous visual cues. Psychovisual experiments, in particular from the Gestaltists [1] indeed show that our vision is structured by a number of grouping laws using different kind of features: contrast change but also direction, spatial proximity, global shape prior or specific structuring pattern like T-junction and X-junction.

One important grouping law among those is the good continuation principle, which states that if two edgels (edge element, i.e. a point together with the orientation of the curve which should pass through it) are not too far apart and share compatible directions, we tend to see the curve to which they are both tangent as an edge. A lot of work has been done towards a digital implementation of that phenomenon [2, 3]. They rely on extracting edges as curves, contrary to pointwise extraction algorithm like the Canny-Deriche filter [4]. To this end, data of different nature needs to be integrated. In particular, to cope with curves one need the orientation (an element of $S^1 = \{v \in \mathbb{R}^2 / \ |v| = 1\}$, the unit circle of $\mathbb{R}^2$) of the tangent to that curve. It is with this idea in mind that this work investigates orientation-driven image processing.

In [5] is presented an axiomatic approach for orientation interpolation in which two operators are singled out: the curvature and the Absolutely Minimizing Lipschitz Extension (AMLE). The second one is well known in scalar computer vision [6] and will allow us here to compute smooth and dense orientation fields that follow the extracted orientation information. We show here how those fields, which capture the geometrical features of the image, can be taken advantage of for orientation driven image processing, namely contour extraction and selective smoothing.

Section 2 briefly presents the AMLE operator for angle data. Application to the extraction of subjective contour via minimal path implemented by Fast Marching is presented in section 3 and selective smoothing of image presenting strong geometrical properties via Line Integral Convolution is described in section 4.

2. EXTENSION OPERATOR ON ANGLE

2.1. Extracting Relevant Points

The first step is to extract a sparse edgel field from the image. The direction of the geometrical structure that passes through a point is given by the vector orthogonal to the gradient, possibly smoothed. Numerous edge point extraction algorithms exist. In the experiments presented here we used the Canny-Deriche filter [4], thresholded so as to keep only the strongest edges.

2.2. Axiomatic of Orientation Interpolation

In [5] an axiomatic approach similar to [6] was developed to find sound interpolation operator by laying out the properties it should follow. It leads to the Absolutely Minimizing Lipschitz Extension (AMLE), a well studied operator for which existence and unicity results are known. ([6] and reference therein). In particular it satisfies a maximality principle which guarantee that the solution is oscillation free.

Let $\Omega \subset \mathbb{R}^2$. Let S^1 be parametrized by angles in $[0, 2\pi[$. Let $D \subset \Omega$ be a set of curves and/or points and $\theta_0 : D \to S^1$. Then $\theta : \Omega \to S^1$ is the AMLE of θ_0 in Ω if:

$$\begin{cases} D^2\theta(D\theta, D\theta) = 0 \text{ in } \Omega, \\ \theta|_D = \theta_0 \text{ on } D, \end{cases} \tag{1}$$

i.e. if the second derivative in the direction of the gradient is null. The mathematical theory only hold if the data θ_0 is not surjective, i.e. if it take value in S^1 minus one point. If it is not the case, singularities are bound to appear, which the operator, looking for Lipschitz solutions, cannot handle. The AMLE is a laminar flow orientation operator, not a turbulent flow one. In practice we are interested in geometric structures, which give rise to regular orientation field. Thus this necessary condition will be locally fulfilled.

This equation has been implemented using the associated evolution problem combined to a finite difference scheme. Due to the specificity of angular data and the iterative nature of such a scheme, a multiresolution procedure was used to initialise the numerical resolution [5]. Typical computational time is about a minute for 512x512 images on Pentium IV 2.5Ghz, and the interpolation process is parameter-less .

In Figure 1(a) the computed field is shown for the Lena image. The extracted points are in black, and the field obtained by extending their associated orientations is visualised via its flow line using LIC [7]. It follows when possible the geometry of the image. As stated, for topological reason, singularities are unavoidable. Thus in absence of clear constraints or if they conflict with each other, chaotic zones akin to turbulence flows are created.

3. EXTRACTING CURVES FROM ORIENTATION FIELD AS GEODESICS

We now are provided with a smooth orientation field θ, extending some extracted data. Let us assume that those initial orientations extracted in section 2.1 can be considered as edgels, i.e. they are tangent to an edge; it is the case in the experiments shown in Fig. 1(a) as they originate from a Canny-Deriche filter. The dense orientation field θ is then expected to be tangent to the full edge which the initial edgels are part of. The edges could then be extracted as the integral curves Γ_x of θ, defined as $\frac{\partial \Gamma_x(\lambda)}{\partial \lambda} = u,\ \Gamma_x(0) = x$, with $u = (\cos\theta, \sin\theta)$ and λ an euclidean parametrisation of Γ_x.

Unfortunately, in practice orientation field θ does not follows exactly the edges, and will only be *close* to being tangent to the edges. Thus the curves of interest cannot be extracted as integral lines, but we just know they will be close to being integral lines. Hence given two points we aim at determining the line *as tangent as possible* to θ that join them. This leads to formulating the problem of extracting edges as the computation of geodesics. Such a formulation is well known in the computer vision field [8, 9].

Using minimal path formulation [9], given two point M_0 and M_1 and vector field u the extraction of the looked after curve Γ_{M_0,M_1} is stated as minimizing a given metric $F(u,\Gamma)$. Let us set $E_{F(u,\Gamma)}(M_0,M_1) = \int_{M_0}^{M_1} F(u,\Gamma)$. The curve Γ_{M_0,M_1} is defined by

$$\Gamma_{M_0,M_1} = \arg \min_{\tilde\Gamma \in \mathcal{C}(M_0,M_1)} E_{F(u,\tilde\Gamma)}(M_0,M_1), \qquad (2)$$

with $\mathcal{C}(M_0,M_1)$ the set of continuous curves from M_0 to M_1. To solve for it, we compute

$$E_{M_0}(x) = \min_{\tilde\Gamma \in \mathcal{C}(M_0,x)} E_{F(u,\tilde\Gamma)}(M_0,x), \quad x \in \Omega.$$

$E_{M_0}(x)$ is a convex function whose only minimum is in M_0, thus it leads to Γ_{M_0,M_1} by a simple gradient descent on E_{M_0} from M_1 to M_0.

To actually compute E_{M_0}, the Fast Marching algorithm [10] is used. It is based on a front propagation of equation

$$\frac{\partial C}{\partial t}(C(\lambda,t)) = \frac{1}{F(u,\Gamma_{M_0,C(\lambda,t)})}\mathbf{n}(C(\lambda,t)), \qquad (3)$$

with C the front, initially an infinitesimal circle around M_0, λ an euclidean parametrization of C and $\mathbf{n}$ the outward normal to the front. $E_{M_0}(x)$ is then defined as the arrival time of the front at each point. Thus at any given time, minimal paths have been computed from M_0 to all the points already visited by the front at that time, they can then be used to further propagate it.

The proposed function F is

$$F(u,\Gamma)(y) = < \frac{\partial\Gamma(\lambda)}{\partial\lambda}(y), u^\perp(y) >^2,$$

with λ an Euclidean parametrisation of Γ, $u^\perp$ the orthogonal of u and $< .,. >$ the usual scalar product of $\mathbb{R}^2$. It is by definition nul if u is tangent to Γ and maximum if it is orthogonal. A similar problem was stated in [11] but the authors proposed a function F that did not depend on the curve but on the propagating front C at the time of it's crossing. This however led to non intrinsic curves which depended on the front propagation process itself. This appears as an undesirable property. This function F can be considered the only parameter for the edge extraction.

In Figures 1(a) and 1(b), the results for real images are displayed. On the Lena image (Fig. 1(a)), using the field computed in section 2, we have recovered the top of the hat and the left jaw, which both were not contrasted enough to be extracted by our point-wise thresholded edge extraction filter. This is a typical case of modal completion, also called illusory contour, where the background and the edge have in part the same color. The Figure 1(b) shows two examples of images with low contrast: in both cases the presence of a structure is clearly recognized while continuous contours or edges are either very faint or not actually present. Thus the use of orientation informations allow us to tackle problem on which method purely based on image intensity would encounter difficulties.

4. SMOOTHING ALONG ORIENTATION

In [12] is described an anisotropic smoothing algorithm based on the Line Integral Convolution (LIC) of Cabral and Leedom [7]. Given an image I_0 and a vector field u, if Γ_x is the integral line of u going through x with λ an euclidean parametrization of Γ_x, the unidimensional heat equation constrained on the integral curves of u,

$$\forall x \in \Omega \quad \frac{\partial I(\Gamma_x(\lambda))}{\partial \lambda}(x) = \frac{\partial^2 I(\Gamma_x(\lambda))}{\partial \lambda^2}(x), \qquad (4)$$

is equivalent to a trace based differential operator on the full image. As it is well known that heat equation is equivalent to a Gaussian convolution, (4) is in turn equivalent to

$$\forall x \in \Omega \quad I(t)(x) = \int_{-\infty}^{+\infty} I_0(\Gamma_x(\lambda)) G_t(\lambda) d\lambda, \qquad (5)$$

with G_t the Gauss function of standard deviation $\sqrt{t}$: $G_t(x) = \frac{1}{\sqrt{4\pi t}} \exp(-\frac{x^2}{4t})$. This is the continuous formulation of the LIC visualization algorithm [7], which is used to visualize the vector field in Fig. 1(a) (left image).

The main issue is that no vector field u is straightforwardly available for a given image. Beside, the extraction of such a field is in the general case not obvious, as images do not necessarily have everywhere strong oriented geometrical features. In [12], the local geometrical information of the image was captured into a diffusion tensor based on the structure tensor [13] which was decomposed into vectors to apply the described LIC-based regularisation.

We have described above a method to compute dense orientation fields as the extension of extracted geometrical structures. While in the general case it would be relevant only on edges (see Sec. 3), in some particular cases it might be an accurate description of the whole image. Images displayed in Fig. 1(c) are accurately described by a set of linear or curvilinear parallel lines. For such images the AMLE orientation field can then be used to implement the LIC-based regularization defined by Eq. 5.

More precisely, for a given image, we proceed as follows. Relevant points are extracted by a thresholded Canny-Deriche filter and the corresponding orientation is computed as the orthogonal to the gradient. The AMLE operator is then applied to obtain a dense orientation field which follows the geometrical features of the image. A Gaussian smoothing constrained to the integral lines as per equation 5 is then applied. Once the initial point are extracted (see section 2.1), the only parameter is the smoothing strength, which is the length of the Gaussian kernel, or equivalently the length on the integral curve.

The image in the left part of figure 1(c) is a patch of sand. This is a natural image with no artificial noise, and the geometry of the image underlying it's grainy natural aspect is reccovered. On the right is a wood texture image with Gaussian noise of standard deviation 20 added. It exhibits vertical cracks, which were captured in the computed orientation field. As illustrated, the orientation based filtering leads to an efficient restoration. Thus, when strong geometrical structures are available, the use of such orientation fields is an elegant and intuitive alternative to the diffusion tensor [12].

5. CONCLUSION

Thanks to the AMLE interpolation operator we were able, given a sparse edgel field, to build a dense orientation field having good mathematical properties in its laminar part. If the edgels represent geometrical data from an image, we showed that this field is a way of recovering geometrical informations. It can be used to extract subjective contours, thus supplying information where the contrast information is low, and smooth strongly geometrical images while fully retaining their structures. While those algorithm are almost parameter-less, they are obviously very dependent on the edgels originally extracted. While the method used here of thresholded Canny-Deriche filter is efficient, more sophisticated method might be investigated, for example by using adaptative multiscale thresholding method or orientation based point-wise extraction algorithm.

In the presented algorithms only the orientation is taken into account. It worked well to take advantage of the geometry of the image in an orientation plane, but both methods could benefit from a coupling with intensity or contrast based information, to extend them to a larger class of image.

6. REFERENCES

[1] G. Kaniza, *La grammaire du voir*, Diderot, 1996.

[2] P. Parent and W. Zucker, “Trace inference, curvature consistency, and curve detection,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 11, no. 8, 1989.

[3] J. Zweck and L.R. Williams, “Euclidian group invariant computation of stochastic completion fields using shiftable-twistable basis function.,” *J. Math. Imaging and Vision*, vol. 21, no. 2, pp. 135–154, 2004.

[4] R. Deriche, “Using canny’s criteria to derive a recursively implemented optimal edge detector,” *Int. J. Computer Vision*, vol. 1, no. 2, pp. 167–187, 1987.

[5] A. Chessel, F. Cao, and R. Fablet, “Orientation interpolation : an axiomatic approach,” Tech. Rep., IRISA, in preparation.

[6] V. Caselles, J.M. Morel, and C. Sbert, “An axiomatic approach to image interpolation,” *IEEE Trans. Image Processing*, vol. 7, no. 3, pp. 376–386, 1998.

[7] B. Cabral and L.C. Leedom, “Imaging vector field using line integral convolution,” *Computer Graphics Proceedings*, pp. 263–270, 1993.

[8] V. Caselles, R. Kimmel, and G. Sapiro, “Geodesic active contours,” *Int. J. Computer Vision*, vol. 22, no. 1, pp. 61–79, 1997.

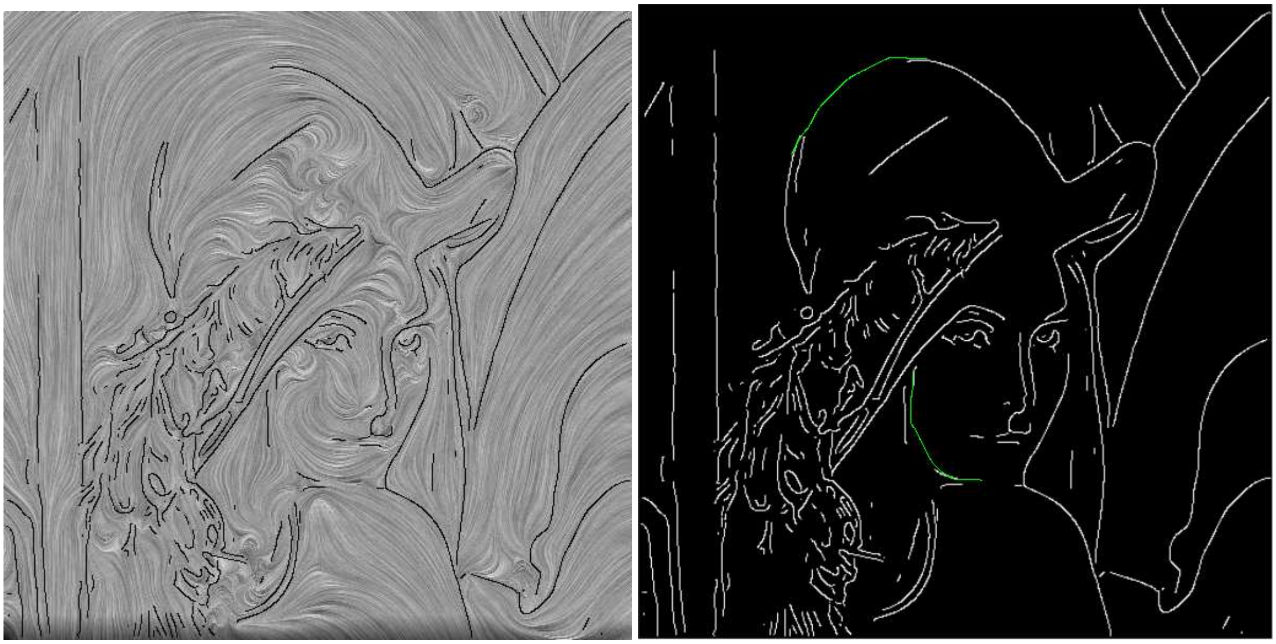
[9] Laurent D. Cohen, “Minimal paths and fast marching methods for image analysis,” in *Mathematical Models in Computer Vision: The Handbook*, Nikos Paragios, Yunmei Chen, and Olivier Faugeras, Eds. Springer, 2005.

[10] J.A. Sethian, *Level Set Methods and Fast Marching Methods*, Cambridge University Press, 1999.

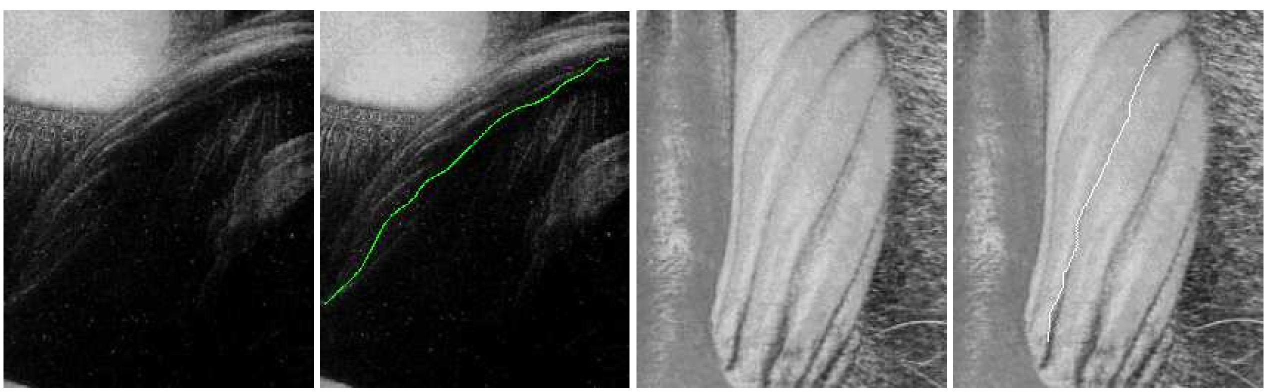
[11] G.J.M. Parker, C.A.M. Wheeler-kingshott, and G.J. Barker, “Estimating distributed anatomical connectivity using fast-marching methods and tensor diffusion imaging,” *IEEE Transaction on Medical Imaging*, vol. 21, no. 5, 2002.

[12] D. Tschumperlé, “LIC-based regularization of multi-valued images,” in *ICIP 05, Genoa, Italy*, 2005.

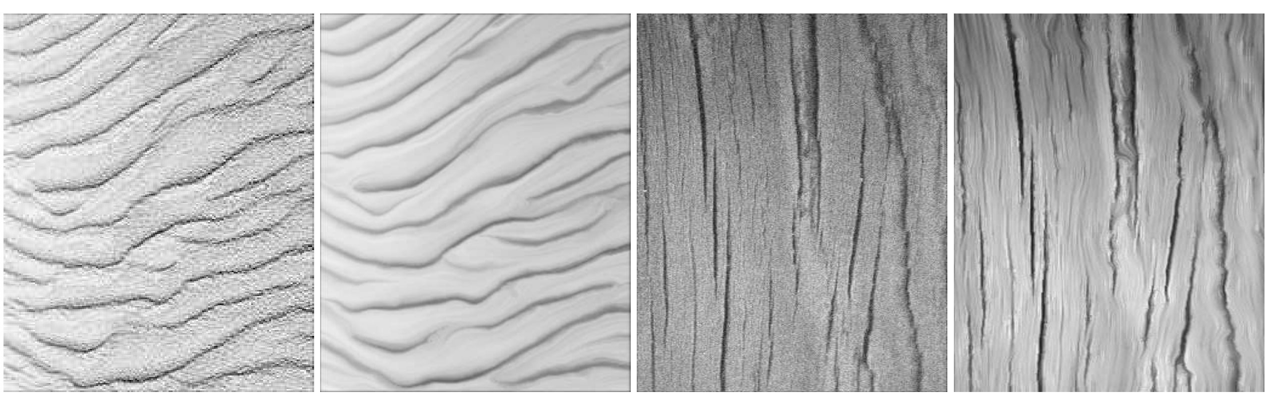
[13] J. Weickert, *Anisotropic Diffusion in Image Processing*, Teubner-Verlag, Stuttgart, 1998.



(a) Left: in black the extracted point by Canny-Deriche on top of the integral line of the field computed by AMLE, right: the same extracted point with the extracted curves in green (top of the hat and left jaw).



(b) Two example of curve extraction in poorly contrasted image part, left Da Vinci's Mona Lisa scarf, right a detail of the baboon image.



(c) Left: sand patch, left original image (no noise added), right LIC-smoothed version using the orientation field computed as per Sec. 2 (not shown), right: wood texture image, left original (with Gaussian noise $\sigma = 20$) and right LIC-smoothed.

Fig. 1.